

Fig.1 – Simple mechanics and their configuration manifolds.

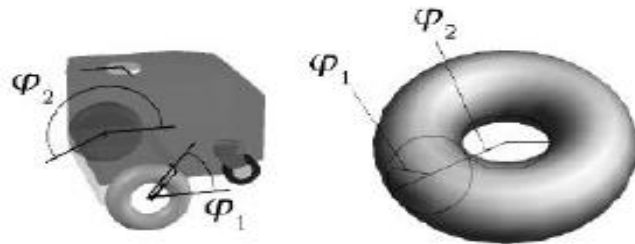
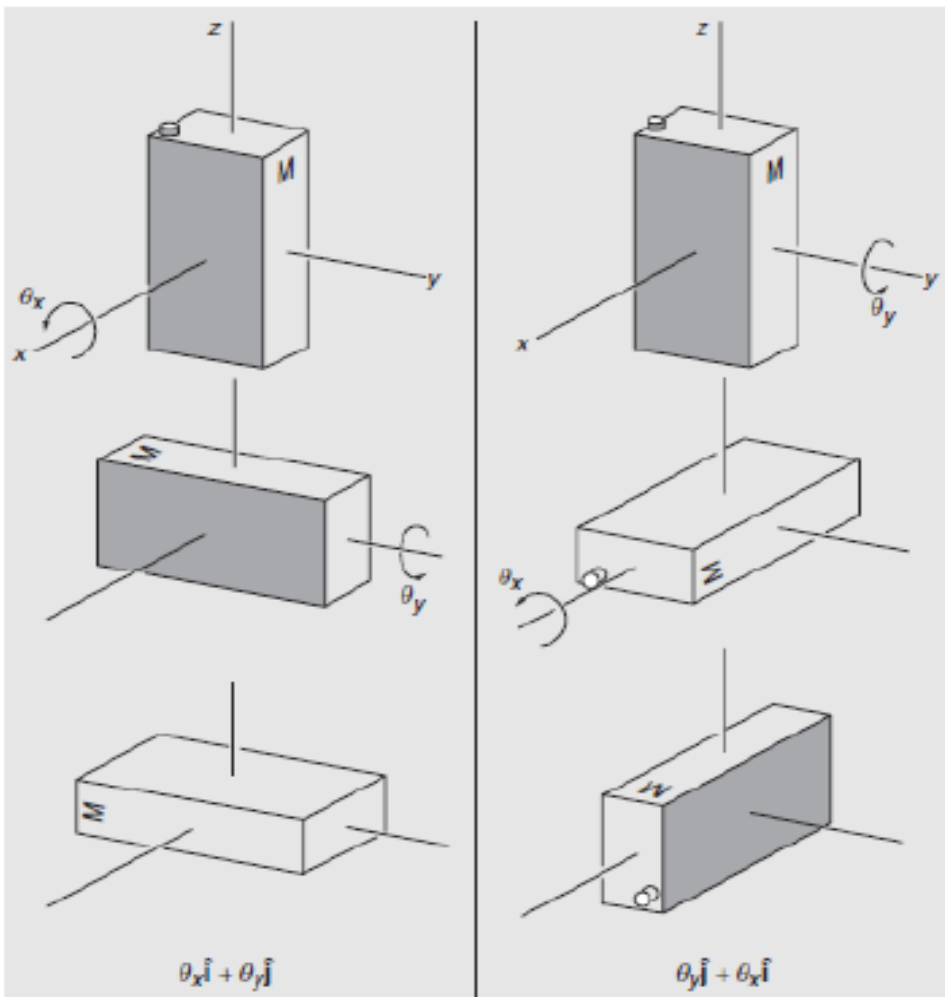


Fig.2 – Part of configuration manifold of mobile robot.



It is evident from the drawings that

$$\theta_x \hat{i} + \theta_y \hat{j} \neq \theta_y \hat{j} + \theta_x \hat{i}.$$

Definition

A manifold of dimension n is a set M which is locally homeomorphic* to R^n .

Homeomorphism:

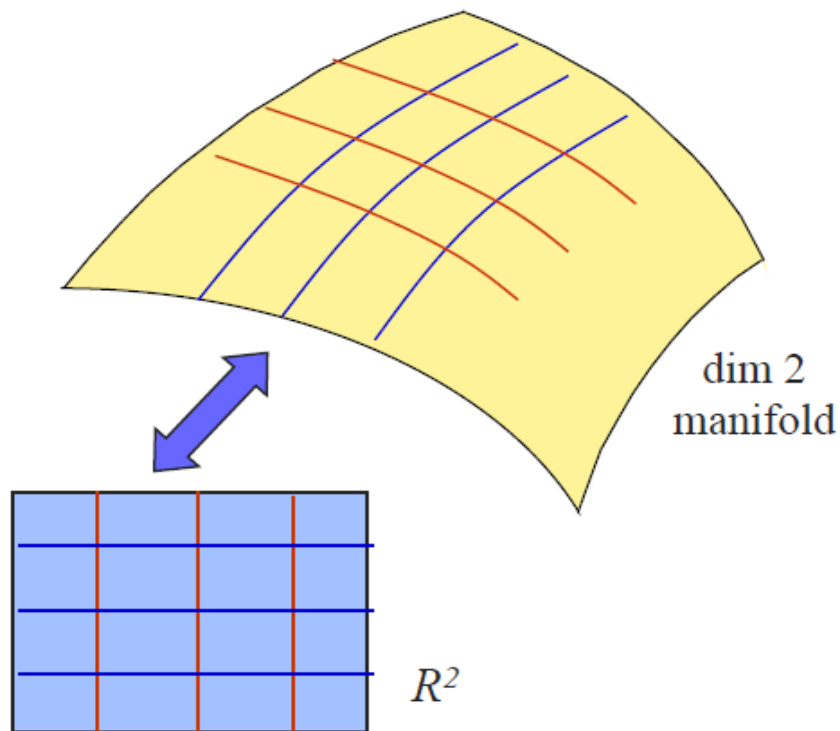
A map f from M to N and its inverse, f^{-1} are both continuous.

Smooth map

A map f from $U \subset R^m$ to $V \subset R^n$ is smooth if all partial derivatives of f , of all orders, exist and are continuous.

Diffeomorphism

A smooth map f from $U \subset R^n$ to $V \subset R^n$ is a diffeomorphism if all partial derivatives of f^{-1} , of all orders, exist and are continuous.



Rigid Body Kinematics

Smooth Manifold

- Differentiable manifold is locally homeomorphic to R^n
- Parametrize the manifold using a set of local coordinate charts
 - ◆ $(U, \phi), (V, \Psi), \dots$

- Require compatibility on overlaps
 C^∞ related

- Collection of charts covering M

